Table 21: WCS and celestial coordinates notation.

Variable(s)	Meaning	Related FITS keywords
i	Index variable for world coordinates	
j	Index variable for pixel coordinates	
a	Alternative WCS version code	
p_j	Pixel coordinates	
r_j	Reference pixel coordinates	CRPIX*ja*
m_{ij}	Linear transformation matrix	CD*i_ja* or PC*i_ja*
s_i	Coordinate scales	CDELT*ia*
(x, y)	Projection plane coordinates	
(ϕ, θ)	Native longitude and latitude	
(α, δ)	Celestial longitude and latitude	
(ϕ_0, θ_0)	Native longitude and latitude of the fiducial point	PV*i*_1*a*† , PV*i*_2*a*†
(α_0, δ_0)	Celestial longitude and latitude of the fiducial point	CRVAL*ia*
(α_p, δ_p)	Celestial longitude and latitude of the native pole	
(ϕ_p, θ_p)	Native longitude and latitude of the celestial pole	LONPOLE*a* (=PV*i*_3*a*†), LATPOLE*a* (=PV*i*_4*a*†)

Notes. † Associated with *longitude* axis *i*.

in the CRPIX*i* keywords, and the world coordinates at the reference point are encoded in the CRVAL*i* keywords. For additional details, see Greisen & Calabretta (2002).

The third step of the process, computing the final world coordinates, depends on the type of coordinate system, which is indicated with the value of the CTYPE*i* keyword. For some simple, linear cases an appropriate choice of normalization for the scale factors allows the world coordinates to be taken directly (or by applying a constant offset) from the x_i (e.g., some spectra). In other cases it is more complicated, and may require the application of some non-linear algorithm (e.g., a projection, as for celestial coordinates), which may require the specification of additional parameters. Where necessary, numeric parameter values for non-linear algorithms *must* be specified via PV*i_m* keywords and character-valued parameters will be specified via PS*i_m* keywords, where *m* is the parameter number.

The application of these formalisms to coordinate systems of interest is discussed in the following sub-sections: Sect. 8.2 describes general WCS representations (see Greisen & Calabretta 2002), Sect. 8.3 describes celestial coordinate systems (see Calabretta & Greisen 2002)), Sect. 8.4 describes spectral coordinate systems (see Greisen et al. 2006), and Sect. 9 describes the representation of time coordinates (see Rots et al. 2015).

8.2. World coordinate system representations

A variety of keywords have been reserved for computing the coordinate values that are to be associated with any pixel location within an array. The full set is given in Table 22; those in most common usage are defined in detail below for convenience. Coordinate system specifications may appear in HDUs that contain simple images in the primary array or in an image extension. Images may also be stored in a multi-dimensional vector cell of a binary table, or as a tabulated list of pixel locations (and optionally, the pixel value) in a table. In these last two types of image representations, the WCS keywords have a different naming convention which reflects the needs of the tabular data structure and the 8-character limit for keyword lengths, but otherwise follow exactly the same rules for type, usage, and default values. See reference Calabretta & Greisen (2002) for example usage of these keywords. All forms of these reserved keywords *must* be used only as specified in this Standard.

In the case of the binary table vector representation, all the images contained in a given column of the table may not necessarily have the same coordinate transformation values. For example, the pixel location of the reference point may be different for each image/row in the table, in which case a single 1CRPn keyword in the header is not sufficient to record the individual value required for each image. In such cases, the keyword must be replaced by a column with the same name (i.e. TTYPEm = '1CRPn') which can then be used to store the pixel location of the reference point appropriate for each row of the table. This convention for expanding a keyword into a table column (or conversely, collapsing a column of identical values into a single header keyword) is commonly known as part of the "Green Bank Convention"[9] for *FITS* keywords. This usage is illustrated in the example header shown in Table 9 of Calabretta & Greisen (2002),

The keywords given below constitute a complete set of fundamental attributes for a WCS description. Although their inclusion in an HDU is optional, *FITS* writers *should* include a complete set of keywords when describing a WCS. In the event that some keywords are missing, default values *must* be assumed, as specified below.

WCSAXES – [integer; default: NAXIS, or larger of WCS indexes *i* or *j*]. Number of axes in the WCS description. This keyword, if present, *must* precede all WCS keywords except NAXIS in the HDU. The value of WCSAXES *may* exceed the number of pixel axes for the HDU.

CTYPE*i* – [character; indexed; default: '␣' (i.e. a linear, undefined axis)]. Type for the intermediate coordinate axis *i*. Any coordinate type that is not covered by this standard or an officially recognized *FITS* convention *shall* be taken to be linear. All non-linear coordinate system names *must* be expressed in '4–3' form: the first four characters specify the coordinate type, the fifth character is a hyphen ('-'), and the remaining three characters specify an algorithm code for computing the world coordinate value. Coordinate types with names of less than four characters are padded on the right with hyphens, and algorithm codes with less than three

[9] Named after a meeting held in Green Bank, West Virginia, USA in 1989 to develop standards for the interchange of single dish radio astronomy data.